

CHAPTER 3 FAULT STUDIES

POWER SYSTEM THREE-PHASE SHORT CIRCUITS

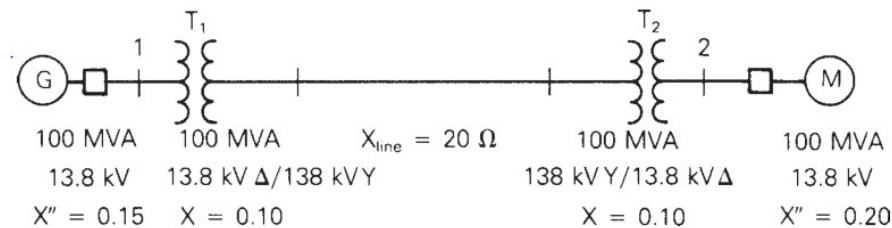
Simplifying assumptions for three-phase short circuit subtransient fault current calculations:

1. Transformers are represented by their leakage reactances. Winding resistances, shunt admittances, and Δ -Y phase shifts are neglected.
2. Transmission lines are represented by their equivalent series reactances. Series resistances and shunt admittances are neglected.
3. Synchronous machines are represented by constant-voltage sources behind subtransient reactances. Armature resistance, saliency, and saturation are neglected.
4. All nonrotating impedance loads are neglected.
5. Induction motors are either neglected (especially for small motors rated less than 50 hp) or represented in the same manner as synchronous machines.

In practice these assumptions should not be made for all cases.

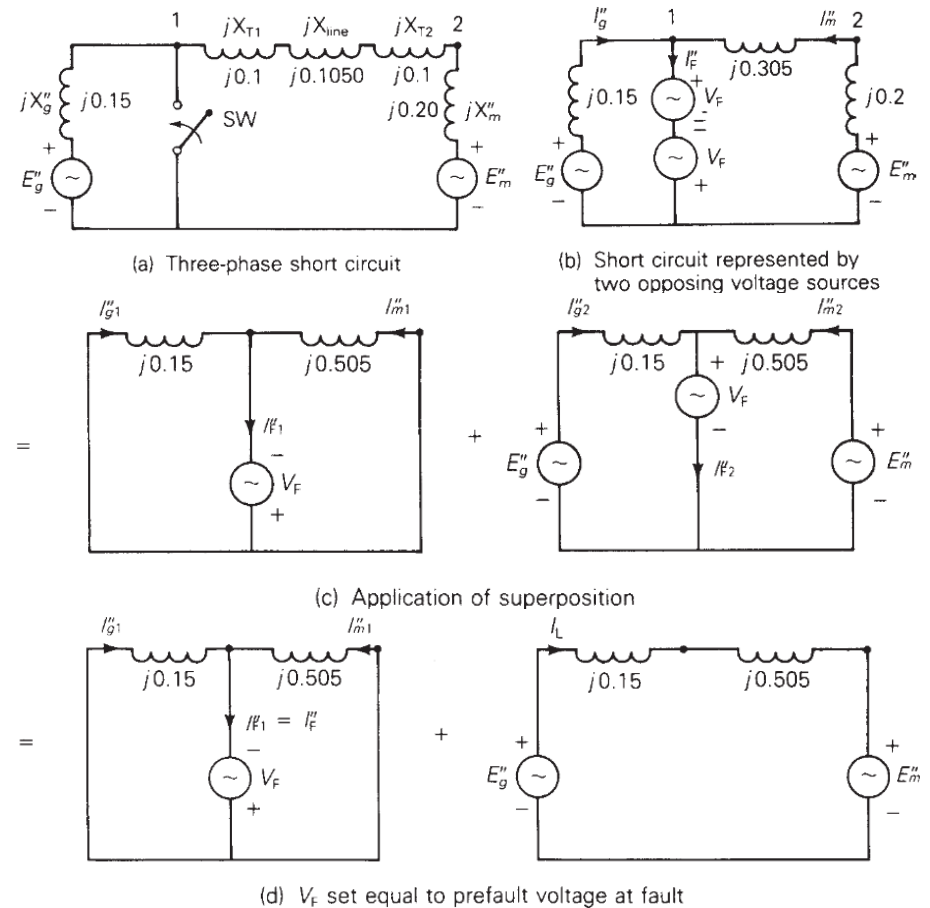
FIGURE 7.3

Single-line diagram of a synchronous generator feeding a synchronous motor



Assume a three-phase fault at bus 1. Superposition is then applied to the power system three-phase short circuit. See Figure 4.

FIGURE 7.4



From Figure 7.4(d), the subtransient fault currents, by superposition, equals the prefault values from the second circuit PLUS the values determined from the first circuit, with V_F set to the prefault value. Neglecting prefault load current, all voltages throughout the second circuit are equal to the prefault voltage, and the prefault voltage is the only source required to calculate the fault current.

Prefault currents are usually small, and hence can be neglected. Otherwise, prefault load currents could be obtained from a power-flow program.

EXAMPLE 7.3 Three-phase short-circuit currents, power system

The synchronous generator in Figure 7.3 is operating at rated MVA, 0.95 p.f. lagging and at 5% above rated voltage when a bolted three-phase short circuit occurs at bus 1. Calculate the per-unit values of (a) subtransient fault current; (b) subtransient generator and motor currents, neglecting prefault current; and (c) subtransient generator and motor currents including prefault current.

SOLUTION

- a. Using a 100-MVA base, the base impedance in the zone of the transmission line is

$$Z_{\text{base, line}} = \frac{(138)^2}{100} = 190.44 \quad \Omega$$

and

$$X_{\text{line}} = \frac{20}{190.44} = 0.1050 \quad \text{per unit}$$

The per-unit reactances are shown in Figure 7.4. From the first circuit in Figure 7.4(d), the Thévenin impedance as viewed from the fault is

$$Z_{\text{Th}} = jX_{\text{Th}} = j \frac{(0.15)(0.505)}{(0.15 + 0.505)} = j0.11565 \quad \text{per unit}$$

and the prefault voltage at the generator terminals is

$$V_F = 1.05 \angle 0^\circ \quad \text{per unit}$$

The subtransient fault current is then

$$I_F'' = \frac{V_F}{Z_{\text{Th}}} = \frac{1.05 \angle 0^\circ}{j0.11565} = -j9.079 \quad \text{per unit}$$

- c. The generator base current is

$$I_{\text{base, gen}} = \frac{100}{(\sqrt{3})(13.8)} = 4.1837 \quad \text{kA}$$

and the prefault generator current is

$$\begin{aligned} I_L &= \frac{100}{(\sqrt{3})(1.05 \times 13.8)} \angle -\cos^{-1} 0.95 = 3.9845 \angle -18.19^\circ \quad \text{kA} \\ &= \frac{3.9845 \angle -18.19^\circ}{4.1837} = 0.9524 \angle -18.19^\circ \\ &= 0.9048 - j0.2974 \quad \text{per unit} \end{aligned}$$

BUS IMPEDANCE MATRIX, Z_{bus}

Subtransient fault currents throughout an N-bus system can be determined from the Z_{bus} and the prefault voltage. Z_{bus} can be computed by first constructing Y_{bus} , via nodal equations, and then inverting Y_{bus} .

Power System Modeling

1. Positive-sequence network used since the power system network is assumed balanced. It remains so under three-phase faults.
2. Lines and transformers are represented by series reactances.
3. Synchronous machines are represented by constant-voltage sources behind subtransient reactances.
4. All resistances, shunt admittances, and nonrotating impedance loads are neglected.

The notes that follows neglect prefault load currents. As before two circuits are used, as in Figure 7.4d. The first is used to find the fault current; both to find the voltage at any bus under fault condition.

Finding the fault currents:

In the first circuit at bus n, the node voltage may be fixed at a value equal to V_F and the voltage source V_F replaced by a source injecting current into the bus, that is:

$$I_n^{(1)} = -I_{Fn}'' \quad E_n^{(1)} = -V_F$$

The superscript (1) denotes the first circuit and the double primes subtransient values. The nodal equation for the first circuit is:

$$Y_{\text{bus}} E^{(1)} = I^{(1)} \quad (7.4.1)$$

or

$$Z_{\text{bus}} I^{(1)} = E^{(1)} \quad (7.4.2)$$

$$Z_{\text{bus}} = Y_{\text{bus}}^{-1} \quad (7.4.3)$$

where Y_{bus} is the positive-sequence bus admittance matrix,

Z_{bus} is the positive-sequence bus impedance matrix,

$E^{(1)}$ is the vector of bus voltages, and

$I^{(1)}$ is the vector of current sources.

Rewriting (7.4.2),

$$\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} & \cdots & Z_{2N} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{n1} & Z_{n2} & \cdots & Z_{nn} & \cdots & Z_{nN} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{Nn} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ -I''_{Fn} \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} E_1^{(1)} \\ E_2^{(1)} \\ \vdots \\ -V_F \\ \vdots \\ E_N^{(1)} \end{bmatrix} \quad (7.4.4)$$

Subtransient fault current:

$$I''_{Fn} = \frac{V_F}{Z_{nn}} \quad (7.4.5)$$

Voltage at any bus k: (superscript (2) denotes the second circuit)

$$E_k^{(1)} = Z_{kn}(-I''_{Fn}) = \frac{-Z_{kn}}{Z_{nn}} V_F \quad (7.4.6)$$

$$E_k^{(2)} = V_F$$

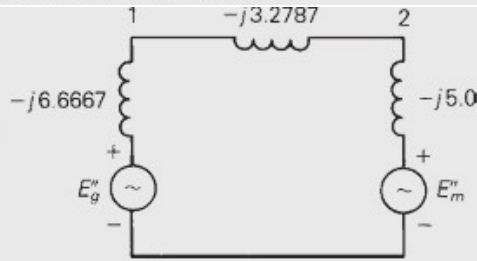
$$\begin{aligned} E_k &= E_k^{(1)} + E_k^{(2)} = \frac{-Z_{kn}}{Z_{nn}} V_F + V_F \\ &= \left(1 - \frac{Z_{kn}}{Z_{nn}}\right) V_F \quad k = 1, 2, \dots, N \end{aligned} \quad (7.4.7)$$

EXAMPLE 7.4 Using Z_{bus} to compute three-phase short-circuit currents in a power system

Faults at bus 1 and 2 in Figure 7.3 are of interest. The prefault voltage is 1.05 per unit and prefault load current is neglected. (a) Determine the 2×2 positive-sequence bus impedance matrix. (b) For a bolted three-phase short circuit at bus 1, use Z_{bus} to calculate the subtransient fault current and the contribution to the fault current from the transmission line. (c) Repeat part (b) for a bolted three-phase short circuit at bus 2.

FIGURE 7.5

Circuit of Figure 7.4(a) showing per-unit admittance values



SOLUTION

a. The circuit of Figure 7.4(a) is redrawn in Figure 7.5 showing per-unit admittance rather than per-unit impedance values. Neglecting prefault load current, $E_g'' = E_m'' = V_F = 1.05/\underline{0^\circ}$ per unit. From Figure 7.5, the positive-sequence bus admittance matrix is

$$Y_{bus} = -j \begin{bmatrix} 9.9454 & -3.2787 \\ -3.2787 & 8.2787 \end{bmatrix} \text{ per unit}$$

Inverting Y_{bus} ,

$$Z_{bus} = Y_{bus}^{-1} = +j \begin{bmatrix} 0.11565 & 0.04580 \\ 0.04580 & 0.13893 \end{bmatrix} \text{ per unit}$$

b. Using (7.4.5) the subtransient fault current at bus 1 is

$$I''_{F1} = \frac{V_F}{Z_{11}} = \frac{1.05/\underline{0^\circ}}{j0.11565} = -j9.079 \text{ per unit}$$

which agrees with the result in Example 7.3, part (a). The voltages at buses 1 and 2 during the fault are, from (7.4.7),

$$E_1 = \left(1 - \frac{Z_{11}}{Z_{11}}\right) V_F = 0$$

$$E_2 = \left(1 - \frac{Z_{21}}{Z_{11}}\right) V_F = \left(1 - \frac{j0.04580}{j0.11565}\right) 1.05/\underline{0^\circ} = 0.6342/\underline{0^\circ}$$

The current to the fault from the transmission line is obtained from the voltage drop from bus 2 to 1 divided by the impedance of the line and transformers T_1 and T_2 :

$$I_{21} = \frac{E_2 - E_1}{j(X_{line} + X_{T1} + X_{T2})} = \frac{0.6342 - 0}{j0.3050} = -j2.079 \text{ per unit}$$

which agrees with the motor current calculated in Example 7.3, part (b), where prefault load current is neglected.

c. Using (7.4.5), the subtransient fault current at bus 2 is

$$I_{F2}'' = \frac{V_F}{Z_{22}} = \frac{1.05/0^\circ}{j0.13893} = -j7.558 \text{ per unit}$$

and from (7.4.7),

$$E_1 = \left(1 - \frac{Z_{12}}{Z_{22}}\right) V_F = \left(1 - \frac{j0.04580}{j0.13893}\right) 1.05/0^\circ = 0.7039/0^\circ$$

$$E_2 = \left(1 - \frac{Z_{22}}{Z_{22}}\right) V_F = 0$$

The current to the fault from the transmission line is

$$I_{12} = \frac{E_1 - E_2}{j(X_{\text{line}} + X_{T1} + X_{T2})} = \frac{0.7039 - 0}{j0.3050} = -j2.308 \text{ per unit}$$

The Rake equivalent:

The same results as above can be obtained from the rake equivalent shown in Figure 7.6. It is based on the network shown below, which is constructed on the assumption that prefault currents are negligible.

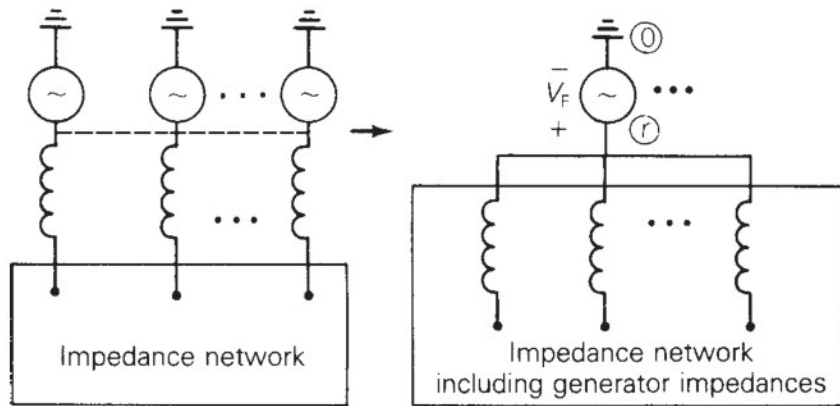
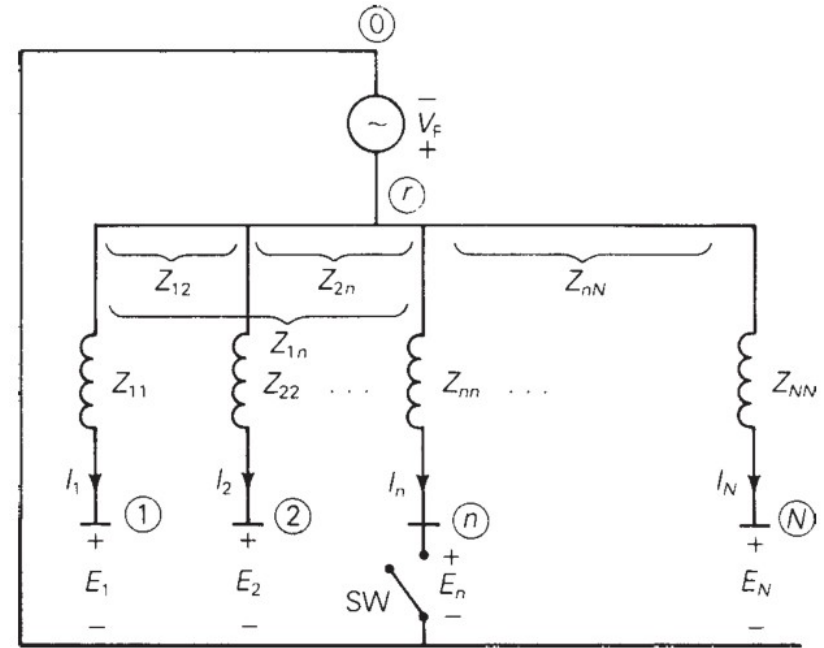


FIGURE 7.6

Bus impedance equivalent circuit (rake equivalent)



Using Z_{bus} , the fault currents in Figure 7.6 are given by

$$\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} & \cdots & Z_{2N} \\ \vdots & \vdots & & \vdots & & \vdots \\ Z_{n1} & Z_{n2} & \cdots & Z_{nn} & \cdots & Z_{nN} \\ \vdots & \vdots & & \vdots & & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{Nn} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} V_F - E_1 \\ V_F - E_2 \\ \vdots \\ V_F - E_n \\ \vdots \\ V_F - E_N \end{bmatrix} \quad (7.4.8)$$

where

$I_1, I_2, \dots$ are the branch currents

$(V_F - E_1); (V_F - E_2) \dots$ are the voltages across the branches

$Z_{11}, Z_{22}, \dots, Z_{NN}$ are the diagonal elements of the Z_{bus} which are the self-impedances. The off-diagonal elements are the mutual impedances.

Pre- fault conditions, neglecting prefault load currents:

Switch SW is open, and all currents are zero and the voltage at each bus with respect to the neutral equals V_F .

Fault conditions, a short circuit at bus n :
Switch SW is closed.

$$E_n = 0 \quad I_{Fn}'' = I_n = V_F / Z_{nn}$$

$$E_k = \frac{-Z_{kn}}{Z_{nn}} V_F + V_F$$

EXAMPLE 7.5

PowerWorld Simulator case Example 7_5 models the 5-bus power system whose one-line diagram is shown in Figure 6.2. Machine, line, and transformer data are given in Tables 7.3, 7.4, and 7.5. This system is initially unloaded. Prefault voltages at all the buses are 1.05 per unit. Use PowerWorld Simulator to determine the fault current for three-phase faults at each of the buses.

TABLE 7.3	
Synchronous machine data for SYMMETRICAL SHORT CIRCUITS program*	Machine Subtransient Reactance— X_d'' (per unit)
	Bus
	1 0.045
	3 0.0225
* $S_{\text{base}} = 100$ MVA $V_{\text{base}} = 15$ kV at buses 1, 3 $= 345$ kV at buses 2, 4, 5	

TABLE 7.4	
Line data for SYMMETRICAL SHORT CIRCUITS program	Equivalent Positive-Sequence Series Reactance (per unit)
	Bus-to-Bus
	2–4 0.1
	2–5 0.05
4–5 0.025	

TABLE 7.5	
Transformer data for SYMMETRICAL SHORT CIRCUITS program	Leakage Reactance— X (per unit)
	Bus-to-Bus
	1–5 0.02
	3–4 0.01

SOLUTION To fault a bus from the one-line, first right-click on the bus symbol to display the local menu, and then select “Fault.” This displays the **Fault** dialog (see Figure 7.8). The selected bus will be automatically selected as the fault location. Verify that the Fault Location is “Bus Fault” and the Fault Type is “3 Phase Balanced” (unbalanced faults are covered in Chapter 9). Then select “**Calculate**,” located in the bottom left corner of the dialog, to determine the fault currents and voltages. The results are shown in the tables at the bottom of the dialog. Additionally, the values can be animated on the one-line by changing the Online Display Field value. Since with a three-phase

fault the system remains balanced, the magnitudes of the a phase, b phase and c phase values are identical. The 5×5 Z_{bus} matrix for this system is shown in Table 7.6, and the fault currents and bus voltages for faults at each of the buses are given in Table 7.7. Note that these fault currents are subtransient fault currents, since the machine reactance input data consist of direct axis subtransient reactances.

TABLE 7.6	
Z_{bus} for Example 7.5	$j \begin{bmatrix} 0.0279725 & 0.0177025 & 0.0085125 & 0.0122975 & 0.020405 \\ 0.0177025 & 0.0569525 & 0.0136475 & 0.019715 & 0.02557 \\ 0.0085125 & 0.0136475 & 0.0182425 & 0.016353 & 0.012298 \\ 0.0122975 & 0.019715 & 0.016353 & 0.0236 & 0.017763 \\ 0.020405 & 0.02557 & 0.012298 & 0.017763 & 0.029475 \end{bmatrix}$

TABLE 7.7				
Fault currents and bus voltages for Example 7.5				
Fault Bus	Fault Current (per unit)	Contributions to Fault Current		
		Gen Line or TRSF	Bus-to-Bus	Current (per unit)
1	37.536	G 1 T 1	GRND–1 5–1	23.332 14.204
2	18.436	L 1 L 2	4–2 5–2	6.864 11.572
3	57.556	G 2 T 2	GRND–3 4–3	46.668 10.888
4	44.456	L 1 L 3 T 2	2–4 5–4 3–4	1.736 10.412 32.308
5	35.624	L 2 L 3 T 1	2–5 4–5 1–5	2.78 16.688 16.152

Per-Unit Bus Voltage Magnitudes during the Fault					
$V_F = 1.05$	Bus 1	Bus 2	Bus 3	Bus 4	Bus 5
Fault Bus:					
1	0.0000	0.7236	0.5600	0.5033	0.3231
2	0.3855	0.0000	0.2644	0.1736	0.1391
3	0.7304	0.7984	0.0000	0.3231	0.6119
4	0.5884	0.6865	0.1089	0.0000	0.4172
5	0.2840	0.5786	0.3422	0.2603	0.0000

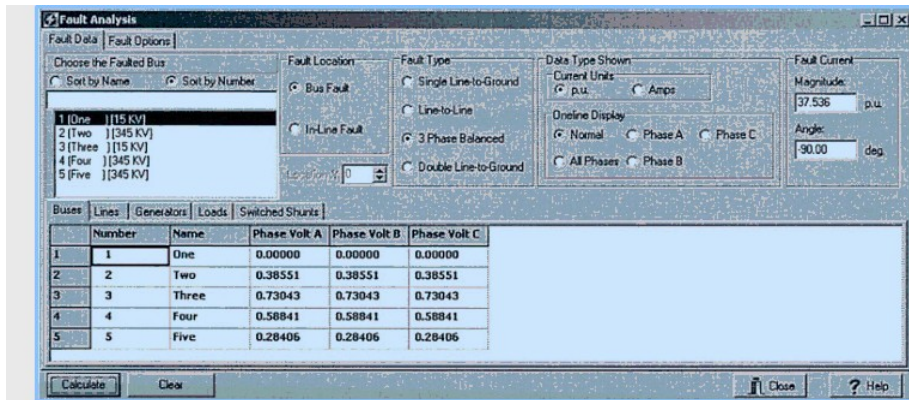


FIGURE 7.8 Fault Analysis Dialog for Example 7.5—fault at bus 1

EXAMPLE 7.6

Redo Example 7.5 with an additional line installed between buses 2 and 4. This line, whose reactance is 0.075 per unit, is not mutually coupled to any other line.

SOLUTION The modified system is contained in PowerWorld Simulator case Example 7_6. Z_{bus} along with the fault currents and bus voltages are shown in Tables 7.8 and 7.9.

TABLE 7.8

Z_{bus} for Example 7.6

$$j \begin{bmatrix} 0.027723 & 0.01597 & 0.00864 & 0.01248 & 0.02004 \\ 0.01597 & 0.04501 & 0.01452 & 0.02097 & 0.02307 \\ 0.00864 & 0.01452 & 0.01818 & 0.01626 & 0.01248 \\ 0.01248 & 0.02097 & 0.01626 & 0.02349 & 0.01803 \\ 0.02004 & 0.02307 & 0.01248 & 0.01803 & 0.02895 \end{bmatrix}$$

TABLE 7.9

Fault currents and bus voltages for Example 7.6

Fault Bus:	Per-Unit Bus Voltage Magnitudes during the Fault				
	Bus 1	Bus 2	Bus 3	Bus 4	Bus 5
1	0.0000	0.6775	0.5510	0.4921	0.3231
2	0.4451	0.0000	0.2117	0.1127	0.2133
3	0.7228	0.7114	0.0000	0.3231	0.5974
4	0.5773	0.5609	0.1109	0.0000	0.3962
5	0.2909	0.5119	0.3293	0.2442	0.0000

Fault Bus	Fault Current (per unit)	Gen Line or TRSF	Bus-to-Bus	Current (per unit)
1	37.872	G 1 T 1	GRND-1 5-1	23.332 14.544
2	23.328	L 1 L 2 L 4	4-2 5-2 4-2	5.608 10.24 7.48
3	57.756	G 2 T 2	GRND-3 4-3	46.668 11.088
4	44.704	L 1 L 3 L 4 T 2	2-4 5-4 2-4 3-4	1.128 9.768 1.504 32.308
5	36.268	L 2 L 3 T 1	2-5 4-5 1-5	4.268 15.848 16.152

FAULT CALCULATIONS USING SYMMETRICAL COMPONENTS

In addition, to the simplifying assumptions for three-phase short circuit subtransient fault current calculations earlier stated, the following other assumptions are also made and used in other types of faults as well:

1. The power system operates under balanced steady-state conditions before the fault occurs. Thus the zero-, positive-, and negative- sequence networks are uncoupled before the fault occurs. During unsymmetrical faults they are interconnected only at the fault location.
2. Prefault load current is neglected. Because of this, the positive- sequence internal voltages of all machines are equal to the pre- fault voltage V_F . Therefore, the prefault voltage at each bus in the positive-sequence network equals V_F .

Sequence Networks

FIGURE 9.1

General three-phase bus

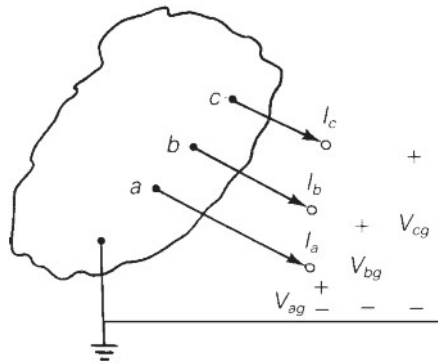
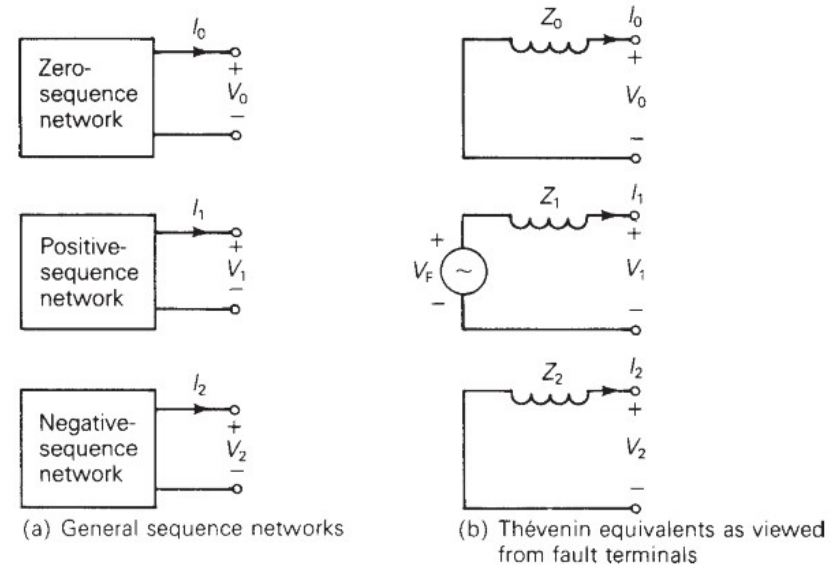


FIGURE 9.2

Sequence networks at a general three-phase bus in a balanced system



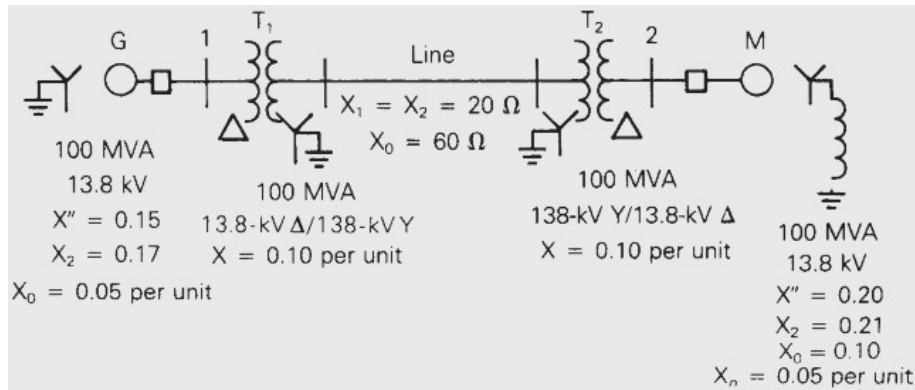
EXAMPLE 9.1

Power-system sequence networks and their Thévenin equivalents

A single-line diagram of the power system considered in Example 7.3 is shown in Figure 9.3, where negative- and zero-sequence reactances are also given. The neutrals of the generator and Δ -Y transformers are solidly grounded. The motor neutral is grounded through a reactance $X_n = 0.05$ per unit on the motor base. (a) Draw the per-unit zero-, positive-, and negative-sequence networks on a 100-MVA, 13.8-kV base in the zone of the generator. (b) Reduce the sequence networks to their Thévenin equivalents, as viewed from bus 2. Prefault voltage is $V_F = 1.05/0^\circ$ per unit. Prefault load current and Δ -Y transformer phase shift are neglected.

FIGURE 9.3

Single-line diagram for Example 9.1



SOLUTION

- a. The sequence networks are shown in Figure 9.4. The positive-sequence network is the same as that shown in Figure 7.4(a). The negative-sequence network is similar to the positive-sequence network, except that there are no sources, and negative-sequence machine reactances are shown. Δ -Y phase shifts are omitted from the positive- and negative-sequence networks for this example. In the zero-sequence network the zero-sequence generator, motor, and transmission-line reactances are shown. Since the motor neutral is grounded through a neutral reactance X_n , $3X_n$ is included in the zero-sequence motor circuit. Also, the zero-sequence Δ -Y transformer models are taken from Figure 8.19.
- b. Figure 9.5 shows the sequence networks reduced to their Thévenin equivalents, as viewed from bus 2. For the positive-sequence equivalent, the Thévenin voltage source is the prefault voltage $V_F = 1.05 \angle 0^\circ$ per unit.

FIGURE 9.4

Sequence networks for Example 9.1

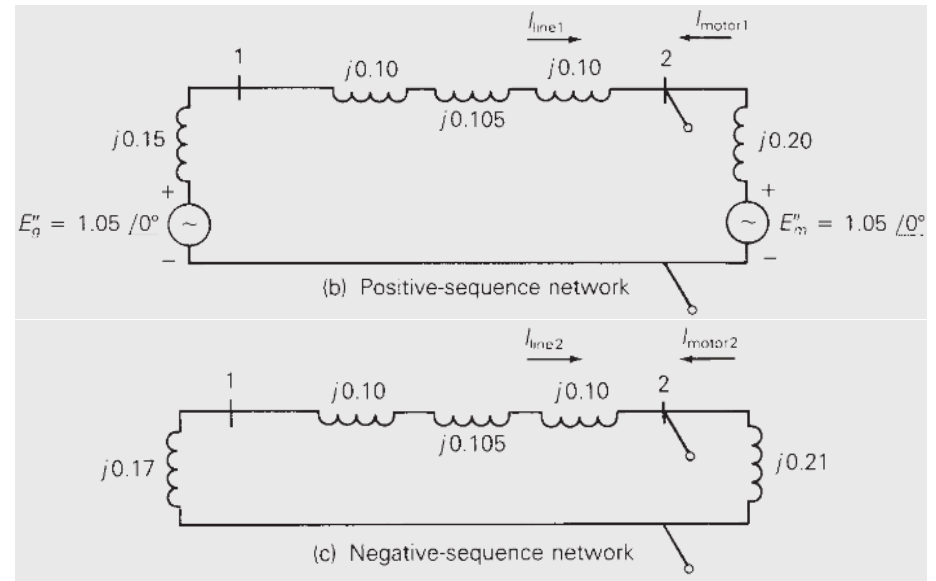
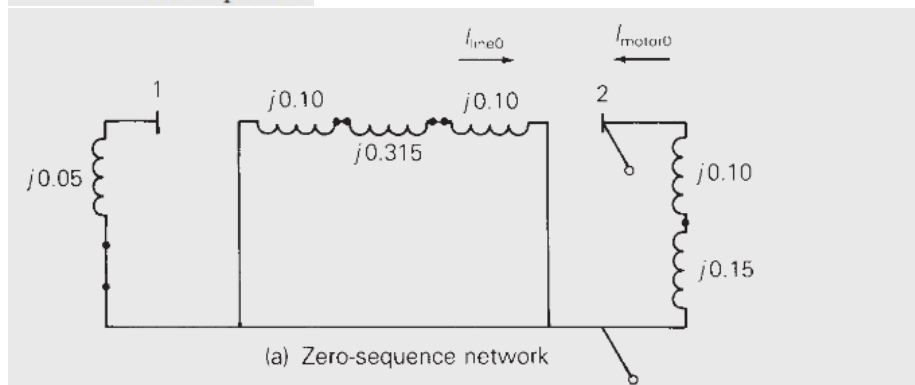
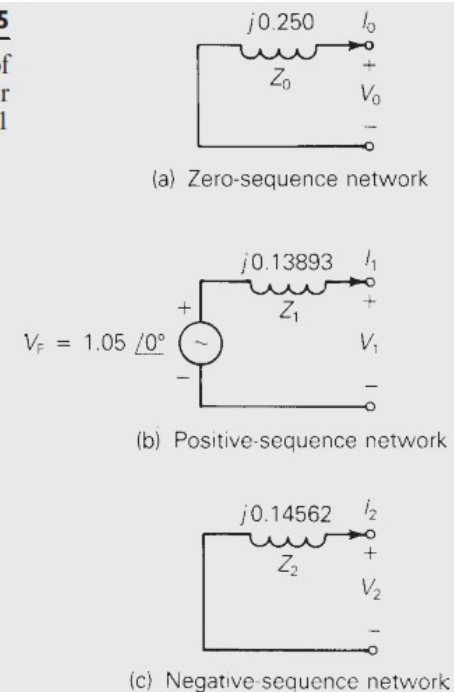


FIGURE 9.5

Thévenin equivalents of sequence networks for Example 9.1



From Figure 9.4, the positive-sequence Thévenin impedance at bus 2 is the motor impedance $j0.20$, as seen to the right of bus 2, in parallel with $j(0.15 + 0.10 + 0.105 + 0.10) = j0.455$, as seen to the left; the parallel combination is $j0.20 // j0.455 = j0.13893$ per unit. Similarly, the negative-sequence Thévenin impedance is $j0.21 // j(0.17 + 0.10 + 0.105 + 0.10) = j0.21 // j0.475 = j0.14562$ per unit. In the zero-sequence network of Figure 9.4, the Thévenin impedance at bus 2 consists only of $j(0.10 + 0.15) = j0.25$ per unit, as seen to the right of bus 2; due to the Δ connection of transformer T_2 , the zero-sequence network looking to the left of bus 2 is open.

EXAMPLE 9.2

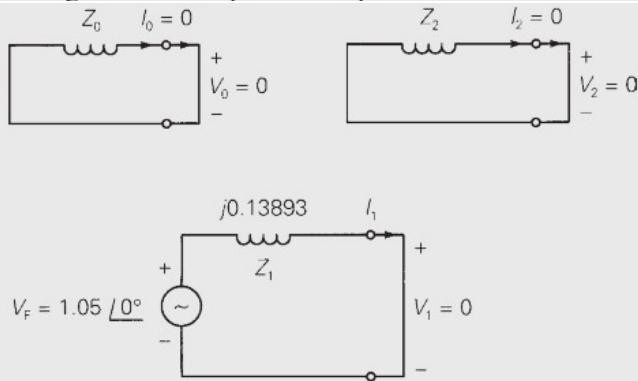
Three-phase short-circuit calculations using sequence networks

Calculate the per-unit subtransient fault currents in phases a , b , and c for a bolted three-phase-to-ground short circuit at bus 2 in Example 9.1.

SOLUTION The terminals of the positive-sequence network in Figure 9.5(b) are shorted, as shown in Figure 9.6. The positive-sequence fault current is

FIGURE 9.6

Example 9.2: Bolted three-phase-to-ground fault at bus 2



$$I_1 = \frac{V_F}{Z_1} = \frac{1.05/0^\circ}{j0.13893} = -j7.558 \text{ per unit}$$

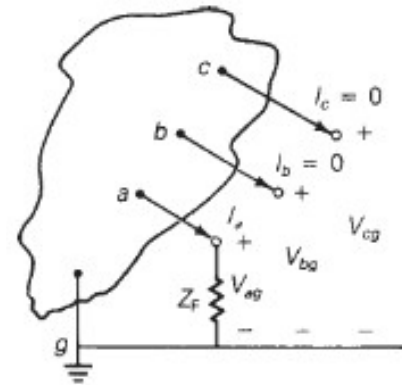
which is the same result as obtained in part (c) of Example 7.4. Note that since subtransient machine reactances are used in Figures 9.4–9.6, the current calculated above is the positive-sequence subtransient fault current at bus 2. Also, the zero-sequence current I_0 and negative-sequence current I_2 are both zero. Therefore, the subtransient fault currents in each phase are, from (8.1.16),

$$\begin{bmatrix} I_a'' \\ I_b'' \\ I_c'' \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j7.558 \\ 0 \end{bmatrix} = \begin{bmatrix} 7.558/-90^\circ \\ 7.558/150^\circ \\ 7.558/30^\circ \end{bmatrix} \text{ per unit}$$

SINGLE LINE-TO-GROUND FAULT

FIGURE 9.7

Single line-to-ground fault

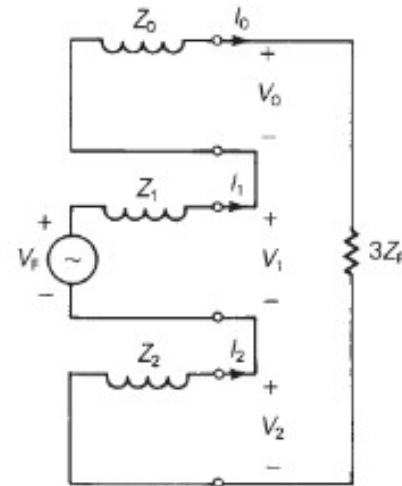


(a) General three-phase bus

Fault conditions in phase domain:

$$V_{ag} = Z_F I_a$$

$$I_b = I_c = 0$$



(b) Interconnected sequence networks

Fault conditions in sequence domain:

$$I_0 = I_1 = I_2$$

$$(V_0 + V_1 + V_2) = 3Z_F I_1$$

EXAMPLE 9.3

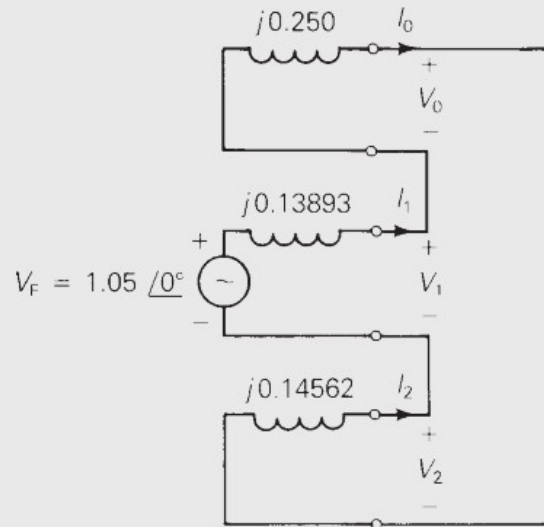
Single line-to-ground short-circuit calculations using sequence networks

Calculate the subtransient fault current in per-unit and in kA for a bolted single line-to-ground short circuit from phase a to ground at bus 2 in Example 9.1. Also calculate the per-unit line-to-ground voltages at faulted bus 2.

SOLUTION The zero-, positive-, and negative-sequence networks in Figure 9.5 are connected in series at the fault terminals, as shown in Figure 9.8.

FIGURE 9.8

Example 9.3: Single line-to-ground fault at bus 2



Since the short circuit is bolted, $Z_F = 0$. From (9.2.7), the sequence currents are:

$$\begin{aligned} I_0 = I_1 = I_2 &= \frac{1.05 \angle 0^\circ}{j(0.25 + 0.13893 + 0.14562)} \\ &= \frac{1.05}{j0.53455} = -j1.96427 \text{ per unit} \end{aligned}$$

From (9.2.8), the subtransient fault current is

$$I_a'' = 3(-j1.96427) = -j5.8928 \text{ per unit}$$

The base current at bus 2 is $100/(13.8\sqrt{3}) = 4.1837 \text{ kA}$. Therefore,

$$I_a'' = (-j5.8928)(4.1837) = 24.65 \angle -90^\circ \text{ kA}$$

From (9.1.1), the sequence components of the voltages at the fault are

$$\begin{aligned} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} &= \begin{bmatrix} 0 \\ 1.05 \angle 0^\circ \\ 0 \end{bmatrix} - \begin{bmatrix} j0.25 & 0 & 0 \\ 0 & j0.13893 & 0 \\ 0 & 0 & j0.14562 \end{bmatrix} \begin{bmatrix} -j1.96427 \\ -j1.96427 \\ -j1.96427 \end{bmatrix} \\ &= \begin{bmatrix} -0.49107 \\ 0.77710 \\ -0.28604 \end{bmatrix} \text{ per unit} \end{aligned}$$

Transforming to the phase domain, the line-to-ground voltages at faulted bus 2 are

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.49107 \\ 0.77710 \\ -0.28604 \end{bmatrix} = \begin{bmatrix} 0 \\ 1.179 \angle 231.3^\circ \\ 1.179 \angle 128.7^\circ \end{bmatrix} \text{ per unit}$$

Note that $V_{ag} = 0$, as specified by the fault conditions. Also $I_b'' = I_c'' = 0$.

Open PowerWorld Simulator case Example 9_3 to see this example. The process for simulating an unsymmetrical fault is almost identical to that for a balanced fault. That is, from the one-line, first right-click on the bus symbol corresponding to the fault location. This displays the local menu. Select “**Fault..**” to display the Fault dialog. Verify that the correct bus is selected, and then set the Fault Type field to “Single Line-to-Ground.” Finally, click on **Calculate** to determine the fault currents and voltages. The results are shown in the tables at the bottom of the dialog. Notice that with an unsymmetrical fault the phase magnitudes are no longer identical. The values can be animated on the one line by changing the Online Display field value, which is shown on the Fault Options page.

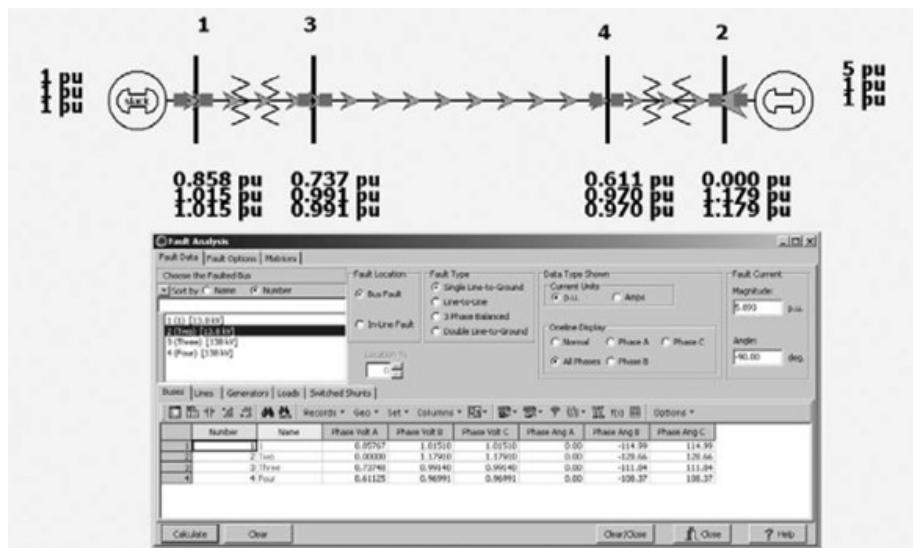
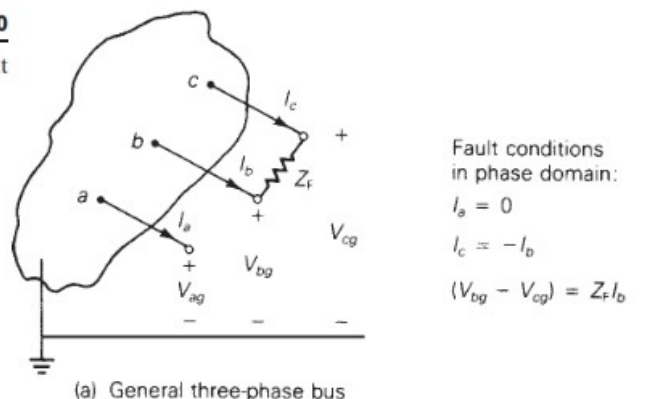


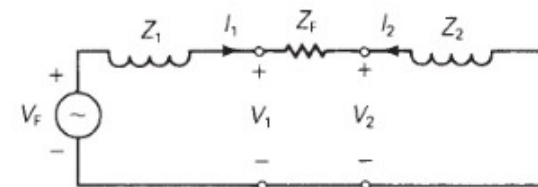
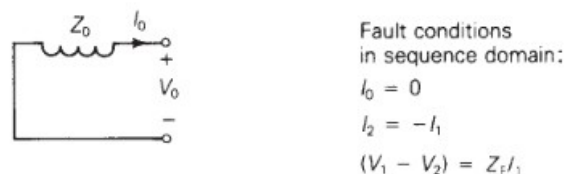
FIGURE 9.9 Screen for Example 9.3—fault at bus 2

LINE-TO-LINE FAULT

FIGURE 9.10
Line-to-line fault



(a) General three-phase bus



(b) Interconnected sequence networks

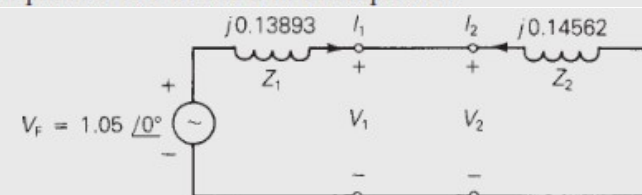
EXAMPLE 9.4

Line-to-line short-circuit calculations using sequence networks

Calculate the subtransient fault current in per-unit and in kA for a bolted line-to-line fault from phase *b* to *c* at bus 2 in Example 9.1.

FIGURE 9.11

Example 9.4: Line-to-line fault at bus 2



SOLUTION The positive- and negative-sequence networks in Figure 9.5 are connected in parallel at the fault terminals, as shown in Figure 9.11. From (9.3.10) with $Z_F = 0$, the sequence fault currents are

$$I_1 = -I_2 = \frac{1.05 \angle 0^\circ}{j(0.13893 + 0.14562)} = 3.690 \angle -90^\circ$$

$$I_0 = 0$$

From (9.3.11), the subtransient fault current in phase *b* is

$$I_b'' = (-j\sqrt{3})(3.690 \angle -90^\circ) = -6.391 = 6.391 \angle 180^\circ \text{ per unit}$$

Using 4.1837 kA as the base current at bus 2,

$$I_b'' = (6.391 \angle 180^\circ)(4.1837) = 26.74 \angle 180^\circ \text{ kA}$$

Also, from (9.3.12) and (9.3.13),

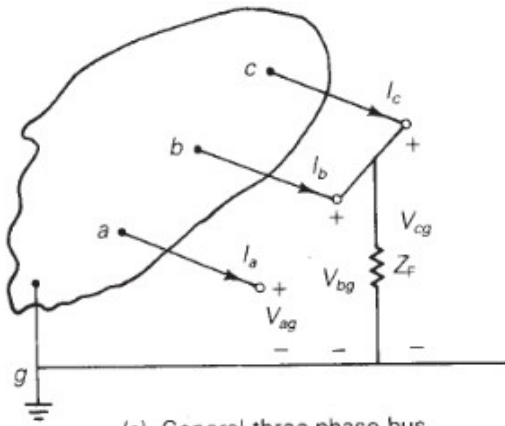
$$I_a'' = 0 \quad I_c'' = 26.74 \angle 0^\circ \text{ kA}$$

The line-to-line fault results for this example can be shown in PowerWorld Simulator by repeating the Example 9.3 procedure, with the exception that the Fault Type field value should be “Line-to-Line.”

DOUBLE LINE-TO-GROUND FAULT

FIGURE 9.12

Double line-to-ground fault

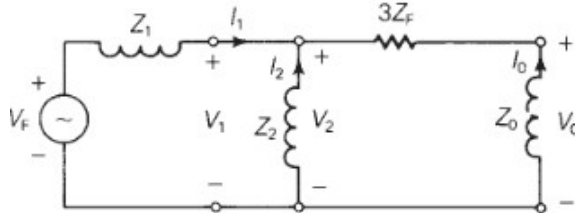


(a) General three-phase bus

Fault conditions in phase domain:

$$I_a = 0$$

$$V_{bg} = V_{cg} = Z_F(I_b + I_c)$$



(b) Interconnected sequence networks

Fault conditions in sequence domain:

$$I_0 + I_1 + I_2 = 0$$

$$V_0 - V_1 = (3Z_F)I_0$$

$$V_1 = V_2$$

EXAMPLE 9.5

Double line-to-ground short-circuit calculations using sequence networks

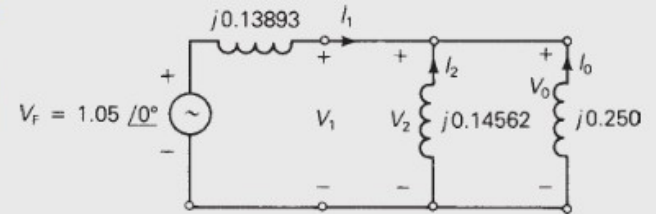
Calculate (a) the subtransient fault current in each phase, (b) neutral fault current, and (c) contributions to the fault current from the motor and from the transmission line, for a bolted double line-to-ground fault from phase *b* to *c* to ground at bus 2 in Example 9.1. Neglect the Δ -Y transformer phase shifts.

SOLUTION

- a. The zero-, positive-, and negative-sequence networks in Figure 9.5 are connected in parallel at the fault terminals in Figure 9.13. From (9.4.12) with $Z_F = 0$,

FIGURE 9.13

Example 9.5: Double line-to-ground fault at bus 2



$$I_1 = \frac{1.05/0^\circ}{j \left[0.13893 + \frac{(0.14562)(0.25)}{0.14562 + 0.25} \right]} = \frac{1.05/0^\circ}{j0.23095}$$

$$= -j4.5464 \text{ per unit}$$

From (9.4.13) and (9.4.14),

$$I_2 = (+j4.5464) \left(\frac{0.25}{0.25 + 0.14562} \right) = j2.8730 \text{ per unit}$$

$$I_0 = (+j4.5464) \left(\frac{0.14562}{0.25 + 0.14562} \right) = j1.6734 \text{ per unit}$$

Transforming to the phase domain, the subtransient fault currents are:

$$\begin{bmatrix} I_a'' \\ I_b'' \\ I_c'' \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} +j1.6734 \\ -j4.5464 \\ +j2.8730 \end{bmatrix} = \begin{bmatrix} 0 \\ 6.8983/158.66^\circ \\ 6.8983/21.34^\circ \end{bmatrix} \text{ per unit}$$

Using the base current of 4.1837 kA at bus 2,

$$\begin{bmatrix} I_a'' \\ I_b'' \\ I_c'' \end{bmatrix} = \begin{bmatrix} 0 \\ 6.8983/158.66^\circ \\ 6.8983/21.34^\circ \end{bmatrix} (4.1837) = \begin{bmatrix} 0 \\ 28.86/158.66^\circ \\ 28.86/21.34^\circ \end{bmatrix} \text{ kA}$$

- b. The neutral fault current is

$$I_n = (I_b'' + I_c'') = 3I_0 = j5.0202 \text{ per unit}$$

$$= (j5.0202)(4.1837) = 21.00/90^\circ \text{ kA}$$

c. Neglecting Δ -Y transformer phase shifts, the contributions to the fault current from the motor and transmission line can be obtained from Figure 9.4. From the zero-sequence network, Figure 9.4(a), the contribution to the zero-sequence fault current from the line is zero, due to the transformer connection. That is,

$$I_{\text{line } 0} = 0$$

$$I_{\text{motor } 0} = I_0 = j1.6734 \text{ per unit}$$

From the positive-sequence network, Figure 9.4(b), the positive terminals of the internal machine voltages can be connected, since $E_g'' = E_m''$. Then, by current division,

$$\begin{aligned} I_{\text{line } 1} &= \frac{X_m''}{X_m'' + (X_g'' + X_{T1} + X_{\text{line } 1} + X_{T2})} I_1 \\ &= \frac{0.20}{0.20 + (0.455)} (-j4.5464) = -j1.3882 \text{ per unit} \end{aligned}$$

$$I_{\text{motor } 1} = \frac{0.455}{0.20 + 0.455} (-j4.5464) = -j3.1582 \text{ per unit}$$

From the negative-sequence network, Figure 9.4(c), using current division,

$$I_{\text{line } 2} = \frac{0.21}{0.21 + 0.475} (j2.8730) = j0.8808 \text{ per unit}$$

$$I_{\text{motor } 2} = \frac{0.475}{0.21 + 0.475} (j2.8730) = j1.9922 \text{ per unit}$$

Transforming to the phase domain with base currents of 0.41837 kA for the line and 4.1837 kA for the motor,

$$\begin{aligned} \begin{bmatrix} I_{\text{line } a}'' \\ I_{\text{line } b}'' \\ I_{\text{line } c}'' \end{bmatrix} &= \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j1.3882 \\ j0.8808 \end{bmatrix} \\ &= \begin{bmatrix} 0.5074/-90^\circ \\ 1.9813/172.643^\circ \\ 1.9813/7.357^\circ \end{bmatrix} \text{ per unit} \\ &= \begin{bmatrix} 0.2123/-90^\circ \\ 0.8289/172.643^\circ \\ 0.8289/7.357^\circ \end{bmatrix} \text{ kA} \end{aligned}$$

$$\begin{bmatrix} I_{\text{motor } a}'' \\ I_{\text{motor } b}'' \\ I_{\text{motor } c}'' \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} j1.6734 \\ -j3.1582 \\ j1.9922 \end{bmatrix}$$

$$\begin{aligned} &= \begin{bmatrix} 0.5074/-90^\circ \\ 4.9986/153.17^\circ \\ 4.9986/26.83^\circ \end{bmatrix} \text{ per unit} \\ &= \begin{bmatrix} 2.123/90^\circ \\ 20.91/153.17^\circ \\ 20.91/26.83^\circ \end{bmatrix} \text{ kA} \end{aligned}$$

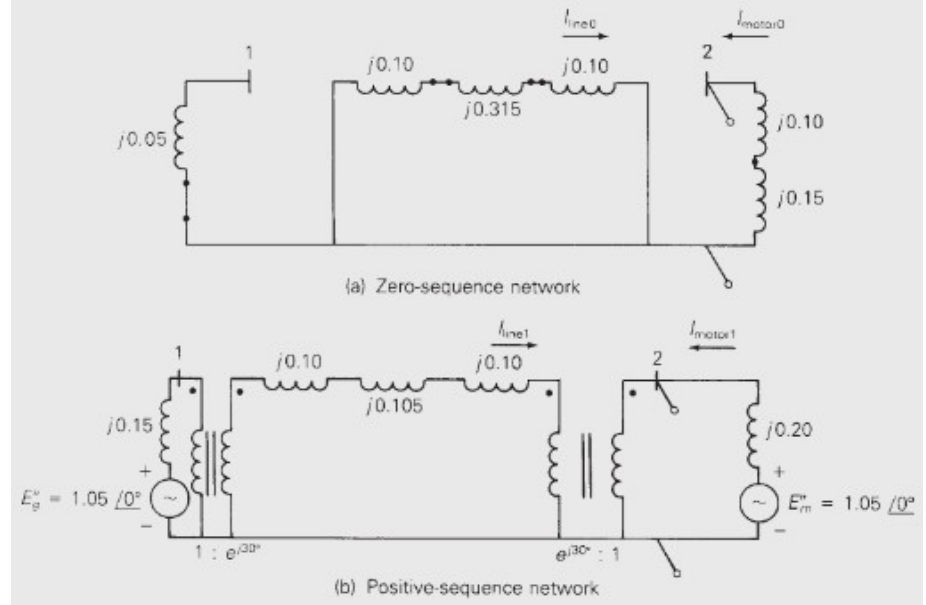
The double line-to-line fault results for this example can be shown in PowerWorld Simulator by repeating the Example 9.3 procedure, with the exception that the Fault Type field value should be "Double Line-to-Ground."

EXAMPLE 9.6

Effect of Δ -Y transformer phase shift on fault currents

Rework Example 9.5, with the Δ -Y transformer phase shifts included. Assume American standard phase shift.

SOLUTION The sequence networks of Figure 9.4 are redrawn in Figure 9.14 with ideal phase-shifting transformers representing Δ -Y phase shifts. In



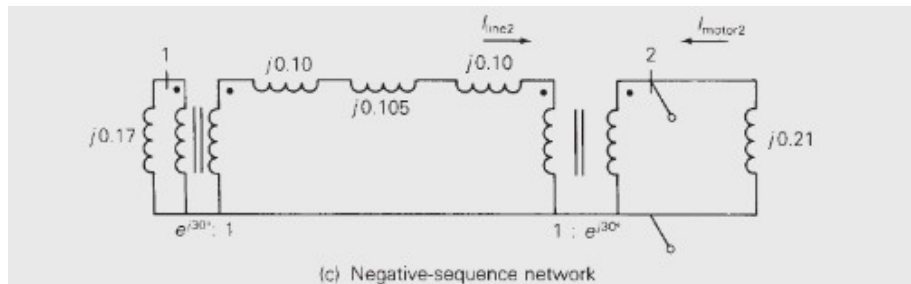


FIGURE 9.14 Sequence networks for Example 9.6

accordance with the American standard, positive-sequence quantities on the high-voltage side of the transformers lead their corresponding quantities on the low-voltage side by 30° . Also, the negative-sequence phase shifts are the reverse of the positive-sequence phase shifts.

- Recall from Section 3.1 and (3.1.26) that per-unit impedance is unchanged when it is referred from one side of an ideal phase-shifting transformer to the other. Accordingly, the Thévenin equivalents of the sequence networks in Figure 9.14, as viewed from fault bus 2, are the same as those given in Figure 9.5. Therefore, the sequence components as well as the phase components of the fault currents are the same as those given in Example 9.5(a).
- The neutral fault current is the same as that given in Example 9.5(b).
- The zero-sequence network, Figure 9.14(a), is the same as that given in Figure 9.4(a). Therefore, the contributions to the zero-sequence fault current from the line and motor are the same as those given in Example 9.5(c).

$$I_{\text{line } 0} = 0 \quad I_{\text{motor } 0} = I_0 = j1.6734 \text{ per unit}$$

The contribution to the positive-sequence fault current from the line in Figure 9.13(b) leads that in Figure 9.4(b) by 30° . That is,

$$I_{\text{line } 1} = (-j1.3882)(1/\underline{30^\circ}) = 1.3882/\underline{-60^\circ} \text{ per unit}$$

$$I_{\text{motor } 1} = -j3.1582 \text{ per unit}$$

Similarly, the contribution to the negative-sequence fault current from the line in Figure 9.14(c) lags that in Figure 9.4(c) by 30° . That is,

$$I_{\text{line } 2} = (j0.8808)(1/\underline{-30^\circ}) = 0.8808/\underline{60^\circ} \text{ per unit}$$

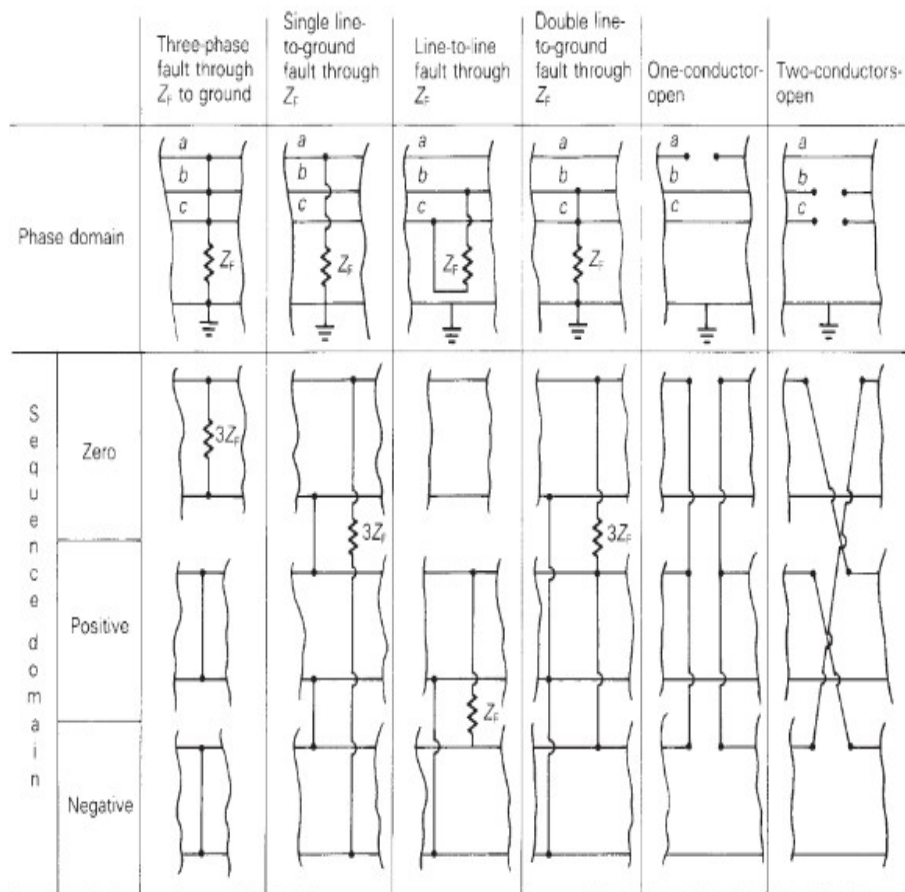
$$I_{\text{motor } 2} = j1.9922 \text{ per unit}$$

Thus, the sequence currents as well as the phase currents from the motor are the same as those given in Example 9.5(c). Also, the sequence currents from the line have the same magnitudes as those given in Example 9.5(c), but the positive- and negative-sequence line currents are shifted by $+30^\circ$ and -30° , respectively. Transforming the line currents to the phase domain:

$$\begin{aligned} \begin{bmatrix} I''_{\text{line } a} \\ I''_{\text{line } b} \\ I''_{\text{line } c} \end{bmatrix} &= \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 1.3882/\underline{-60^\circ} \\ 0.8808/\underline{60^\circ} \end{bmatrix} \\ &= \begin{bmatrix} 1.2166/\underline{-21.17^\circ} \\ 2.2690/\underline{180^\circ} \\ 1.2166/\underline{21.17^\circ} \end{bmatrix} \text{ per unit} \\ &= \begin{bmatrix} 0.5090/\underline{-21.17^\circ} \\ 0.9492/\underline{180^\circ} \\ 0.5090/\underline{21.17^\circ} \end{bmatrix} \text{ kA} \end{aligned}$$

In conclusion, Δ -Y transformer phase shifts have no effect on the fault currents and no effect on the contribution to the fault currents on the fault side of the Δ -Y transformers. However, on the other side of the Δ -Y transformers, the positive- and negative-sequence components of the contributions to the fault currents are shifted by $\pm 30^\circ$, which affects both the magnitude as well as the angle of the phase components of these fault contributions for unsymmetrical faults. ■

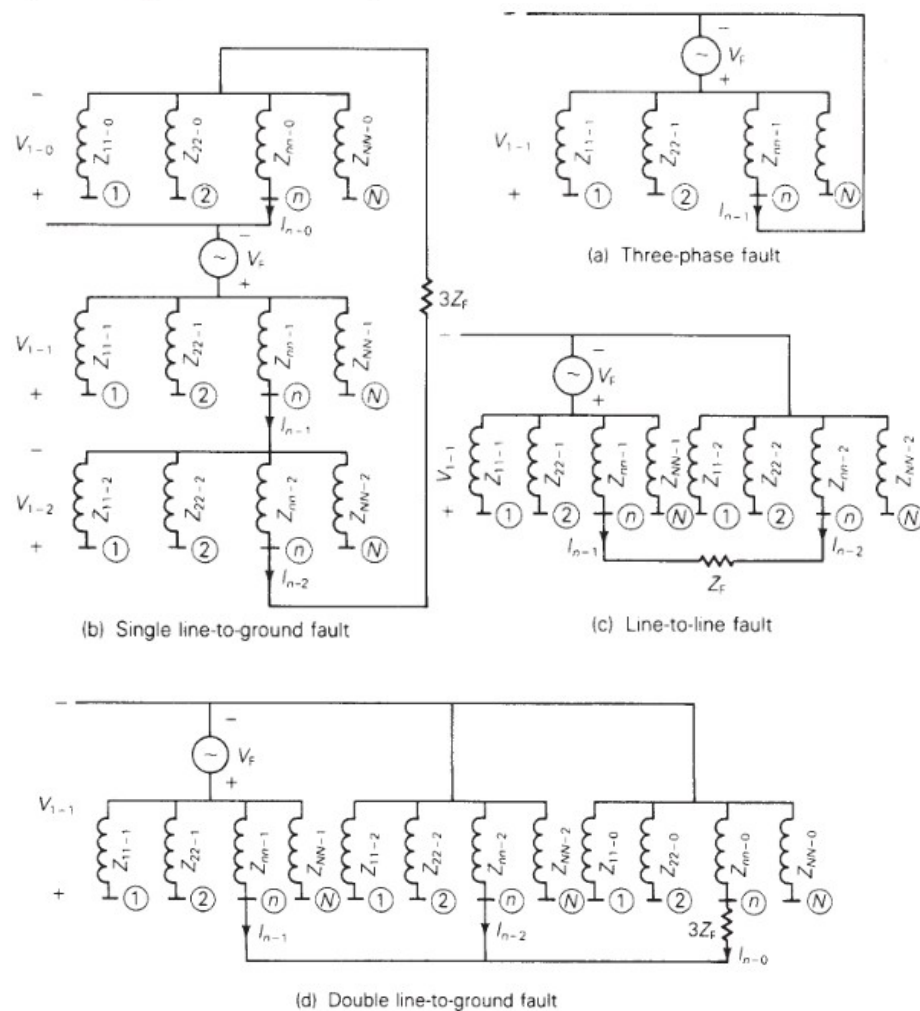
FIGURE 9.15 Summary of faults



SEQUENCE BUS IMPEDANCE MATRICES

FIGURE 9.16

Connection of rake equivalent sequence networks for three-phase system faults (mutual impedances not shown)



Balanced three-phase fault:

$$I_{n-1} = \frac{V_F}{Z_{nn-1}} \quad (9.5.1)$$

$$I_{n-0} = I_{n-2} = 0 \quad (9.5.2)$$

Single line-to-ground fault (phase a to ground):

$$I_{n-0} = I_{n-1} = I_{n-2} = \frac{V_F}{Z_{nn-0} + Z_{nn-1} + Z_{nn-2} + 3Z_F} \quad (9.5.3)$$

Line-to-line fault (phase b to c):

$$I_{n-1} = -I_{n-2} = \frac{V_F}{Z_{nn-1} + Z_{nn-2} + Z_F} \quad (9.5.4)$$

$$I_{n-0} = 0 \quad (9.5.5)$$

Double line-to-ground fault (phase b to c to ground):

$$I_{n-1} = \frac{V_F}{Z_{nn-1} + \left[\frac{Z_{nn-2}(Z_{nn-0} + 3Z_F)}{Z_{nn-2} + Z_{nn-0} + 3Z_F} \right]} \quad (9.5.6)$$

$$I_{n-2} = (-I_{n-1}) \left(\frac{Z_{nn-0} + 3Z_F}{Z_{nn-0} + 3Z_F + Z_{nn-2}} \right) \quad (9.5.7)$$

$$I_{n-0} = (-I_{n-1}) \left(\frac{Z_{nn-2}}{Z_{nn-0} + 3Z_F + Z_{nn-2}} \right) \quad (9.5.8)$$

Also from Figure 9.16, the sequence components of the line-to-ground voltages at any bus k during a fault at bus n are:

$$\begin{bmatrix} V_{k-0} \\ V_{k-1} \\ V_{k-2} \end{bmatrix} = \begin{bmatrix} 0 \\ V_F \\ 0 \end{bmatrix} - \begin{bmatrix} Z_{kn-0} & 0 & 0 \\ 0 & Z_{kn-1} & 0 \\ 0 & 0 & Z_{kn-2} \end{bmatrix} \begin{bmatrix} I_{n-0} \\ I_{n-1} \\ I_{n-2} \end{bmatrix} \quad (9.5.9)$$

If bus k is on the unfaulted side of a Δ -Y transformer, then the phase angles of V_{k-1} and V_{k-2} in (9.5.9) are modified to account for Δ -Y phase shifts. Also, the above sequence fault currents and sequence voltages can be transformed to the phase domain via (8.1.16) and (8.1.9).

EXAMPLE 9.7**Single line-to-ground short-circuit calculations using $Z_{\text{bus } 0}$, $Z_{\text{bus } 1}$, and $Z_{\text{bus } 2}$**

Faults at buses 1 and 2 for the three-phase power system given in Example 9.1 are of interest. The prefault voltage is 1.05 per unit. Prefault load current is neglected. (a) Determine the per-unit zero-, positive-, and negative-sequence bus impedance matrices. Find the subtransient fault current in per-unit for a bolted single line-to-ground fault current from phase a to ground (b) at bus 1 and (c) at bus 2. Find the per-unit line-to-ground voltages at (d) bus 1 and (e) bus 2 during the single line-to-ground fault at bus 1.

SOLUTION

a. Referring to Figure 9.4(a), the zero-sequence bus admittance matrix is

$$Y_{\text{bus } 0} = -j \left[\begin{array}{c|c} 20 & 0 \\ \hline 0 & 4 \end{array} \right] \text{ per unit}$$

Inverting $Y_{\text{bus } 0}$,

$$Z_{\text{bus } 0} = j \left[\begin{array}{c|c} 0.05 & 0 \\ \hline 0 & 0.25 \end{array} \right] \text{ per unit}$$

Note that the transformer leakage reactances and the zero-sequence transmission-line reactance in Figure 9.4(a) have no effect on $Z_{\text{bus } 0}$. The transformer Δ connections block the flow of zero-sequence current from the transformers to bus 1 and 2.

The positive-sequence bus admittance matrix, from Figure 9.4(b), is

$$Y_{\text{bus } 1} = -j \begin{bmatrix} 9.9454 & -3.2787 \\ -3.2787 & 8.2787 \end{bmatrix} \text{ per unit}$$

Inverting $Y_{\text{bus } 1}$,

$$Z_{\text{bus } 1} = j \begin{bmatrix} 0.11565 & 0.04580 \\ 0.04580 & 0.13893 \end{bmatrix} \text{ per unit}$$

Similarly, from Figure 9.4(c)

$$Y_{\text{bus } 2} = -j \begin{bmatrix} 9.1611 & -3.2787 \\ -3.2787 & 8.0406 \end{bmatrix}$$

Inverting $Y_{\text{bus } 2}$,

$$Z_{\text{bus } 2} = j \begin{bmatrix} 0.12781 & 0.05212 \\ 0.05212 & 0.14562 \end{bmatrix} \text{ per unit}$$

b. From (9.5.3), with $n = 1$ and $Z_F = 0$, the sequence fault currents are

$$\begin{aligned} I_{1-0} = I_{1-1} = I_{1-2} &= \frac{V_F}{Z_{11-0} + Z_{11-1} + Z_{11-2}} \\ &= \frac{1.05/0^\circ}{j(0.05 + 0.11565 + 0.12781)} = \frac{1.05}{j0.29346} = -j3.578 \text{ per unit} \end{aligned}$$

The subtransient fault currents at bus 1 are, from (8.1.16),

$$\begin{bmatrix} I''_{1a} \\ I''_{1b} \\ I''_{1c} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -j3.578 \\ -j3.578 \\ -j3.578 \end{bmatrix} = \begin{bmatrix} -j10.73 \\ 0 \\ 0 \end{bmatrix} \text{ per unit}$$

c. Again from (9.5.3), with $n = 2$ and $Z_F = 0$,

$$\begin{aligned} I_{2-0} = I_{2-1} = I_{2-2} &= \frac{V_F}{Z_{22-0} + Z_{22-1} + Z_{22-2}} \\ &= \frac{1.05/0^\circ}{j(0.25 + 0.13893 + 0.14562)} = \frac{1.05}{j0.53455} \\ &= -j1.96427 \text{ per unit} \end{aligned}$$

and

$$\begin{bmatrix} I''_{2a} \\ I''_{2b} \\ I''_{2c} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -j1.96427 \\ -j1.96427 \\ -j1.96427 \end{bmatrix} = \begin{bmatrix} -j5.8928 \\ 0 \\ 0 \end{bmatrix} \text{ per unit}$$

This is the same result as obtained in Example 9.3.

d. The sequence components of the line-to-ground voltages at bus 1 during the fault at bus 1 are, from (9.5.9), with $k = 1$ and $n = 1$,

$$\begin{bmatrix} V_{1-0} \\ V_{1-1} \\ V_{1-2} \end{bmatrix} = \begin{bmatrix} 0 \\ 1.05/0^\circ \\ 0 \end{bmatrix} - \begin{bmatrix} j0.05 & 0 & 0 \\ 0 & j0.11565 & 0 \\ 0 & 0 & j0.12781 \end{bmatrix} \begin{bmatrix} -j3.578 \\ -j3.578 \\ -j3.578 \end{bmatrix}$$

$$= \begin{bmatrix} -0.1789 \\ 0.6362 \\ -0.4573 \end{bmatrix} \text{ per unit}$$

and the line-to-ground voltages at bus 1 during the fault at bus 1 are

$$\begin{bmatrix} V_{1-ag} \\ V_{1-bg} \\ V_{1-cg} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.1789 \\ +0.6362 \\ -0.4573 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0.9843/254.2^\circ \\ 0.9843/105.8^\circ \end{bmatrix} \text{ per unit}$$

- e. The sequence components of the line-to-ground voltages at bus 2 during the fault at bus 1 are, from (9.5.9), with $k = 2$ and $n = 1$,

$$\begin{bmatrix} V_{2-0} \\ V_{2-1} \\ V_{2-2} \end{bmatrix} = \begin{bmatrix} 0 \\ 1.05/0^\circ \\ 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & j0.04580 & 0 \\ 0 & 0 & j0.05212 \end{bmatrix} \begin{bmatrix} -j3.578 \\ -j3.578 \\ -j3.578 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0.8861 \\ -0.18649 \end{bmatrix} \text{ per unit}$$

Note that since both bus 1 and 2 are on the low-voltage side of the Δ -Y transformers in Figure 9.3, there is no shift in the phase angles of these sequence voltages. From the above, the line-to-ground voltages at bus 2 during the fault at bus 1 are

$$\begin{bmatrix} V_{2-ag} \\ V_{2-bg} \\ V_{2-cg} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.8861 \\ -0.18649 \end{bmatrix}$$

$$= \begin{bmatrix} 0.70 \\ 0.9926/249.4^\circ \\ 0.9926/110.6^\circ \end{bmatrix} \text{ per unit}$$

EXAMPLE 9.8

PowerWorld Simulator

Consider the five-bus power system whose single-line diagram is shown in Figure 6.2. Machine, line, and transformer data are given in Tables 9.1, 9.2, and 9.3. Note that the neutrals of both transformers and generator 1 are solidly grounded, as indicated by a neutral reactance of zero for these equipments. However, a neutral reactance = 0.0025 per unit is connected to the generator 2 neutral. The prefault voltage is 1.05 per unit. Using PowerWorld Simulator, determine the fault currents and voltages for a bolted single line-to-ground fault at bus 1, then bus 2, and so on to bus 5.

SOLUTION Open PowerWorld Simulator case Example 9.8 to see this example. Tables 9.4 and 9.5 summarize the PowerWorld Simulator results for each of the faults. Note that these fault currents are subtransient currents, since the machine positive-sequence reactance input consists of direct axis subtransient reactances.

TABLE 9.1

Synchronous machine data for Example 9.8

Bus	X_0 per unit	$X_1 = X_d''$ per unit	X_2 per unit	Neutral Reactance X_n per unit
1	0.0125	0.045	0.045	0
3	0.005	0.0225	0.0225	0.0025

TABLE 9.2

Line data for Example 9.8

Bus-to-Bus	X_0 per unit	X_1 per unit
2-4	0.3	0.1
2-5	0.15	0.05
4-5	0.075	0.025

TABLE 9.3

Transformer data for Example 9.8

Low-Voltage (connection) bus	High-Voltage (connection) bus	Leakage Reactance per unit	Neutral Reactance per unit
1 (Δ)	5 (Y)	0.02	0
3 (Δ)	4 (Y)	0.01	0

$$S_{\text{base}} = 100 \text{ MVA}$$

$$V_{\text{base}} = \begin{cases} 15 \text{ kV at buses 1, 3} \\ 345 \text{ kV at buses 2, 4, 5} \end{cases}$$

TABLE 9.4

Fault currents for
Example 9.8

Fault Bus	Single Line-to-Ground Fault Current (Phase A) per unit/degrees	Contributions to Fault Current				
		GEN LINE OR TRSF	Bus-to- Bus	Phase A	Current Phase B	Phase C
				per unit/degrees		
1	46.02/−90.00	G1	GRND−1	34.41/ −90.00	5.804/ −90.00	5.804/ −90.00
		T1	5−1	11.61/ −90.00	5.804/ 90.00	5.804/ 90.00
2	14.14/−90.00	L1	4−2	5.151/ −90.00	0.1124/ 90.00	0.1124/ 90.00
		L2	5−2	8.984/ −90.00	0.1124/ −90.00	0.1124/ −90.00
3	64.30/−90.00	G2	GRND−3	56.19/ −90.00	4.055/ −90.00	4.055/ −90.00
		T2	4−3	8.110/ −90.00	4.055/ 90.00	4.055/ 90.00
4	56.07/−90.00	L1	2−4	1.742/ −90.00	0.4464/ 90.00	0.4464/ 90.00
		L3	5−4	10.46/ −90.00	2.679/ 90.00	2.679/ 90.00
		T2	3−4	43.88/ −90.00	3.125/ −90.00	3.125/ −90.00
5	42.16/−90.00	L2	2−5	2.621/ −90.00	0.6716/ 90.00	0.6716/ 90.00
		L3	4−5	15.72/ −90.00	4.029/ 90.00	4.029/ 90.00
		T1	1−5	23.82/ −90.00	4.700/ −90.00	4.700/ −90.00

TABLE 9.5

Bus voltages for
Example 9.8

$V_{\text{prefault}} = 1.05 \angle 0$		Bus Voltages during Fault		
Fault Bus	Bus	Phase A	Phase B	Phase C
1	1	0.0000 $\angle$ 0.00	0.9537 $\angle$ −107.55	0.9537 $\angle$ 107.55
	2	0.5069 $\angle$ 0.00	0.9440 $\angle$ −105.57	0.9440 $\angle$ 105.57
	3	0.7888 $\angle$ 0.00	0.9912 $\angle$ −113.45	0.9912 $\angle$ 113.45
	4	0.6727 $\angle$ 0.00	0.9695 $\angle$ −110.30	0.9695 $\angle$ 110.30
	5	0.4239 $\angle$ 0.00	0.9337 $\angle$ −103.12	0.9337 $\angle$ 103.12
2	1	0.8832 $\angle$ 0.00	1.0109 $\angle$ −115.90	1.0109 $\angle$ 115.90
	2	0.0000 $\angle$ 0.00	1.1915 $\angle$ −130.26	1.1915 $\angle$ 130.26
	3	0.9214 $\angle$ 0.00	1.0194 $\angle$ −116.87	1.0194 $\angle$ 116.87
	4	0.8435 $\angle$ 0.00	1.0158 $\angle$ −116.47	1.0158 $\angle$ 116.47
	5	0.7562 $\angle$ 0.00	1.0179 $\angle$ −116.70	1.0179 $\angle$ 116.70
3	1	0.6851 $\angle$ 0.00	0.9717 $\angle$ −110.64	0.9717 $\angle$ 110.64
	2	0.4649 $\angle$ 0.00	0.9386 $\angle$ −104.34	0.9386 $\angle$ 104.34
	3	0.0000 $\angle$ 0.00	0.9942 $\angle$ −113.84	0.9942 $\angle$ 113.84
	4	0.3490 $\angle$ 0.00	0.9259 $\angle$ −100.86	0.9259 $\angle$ 100.86
	5	0.5228 $\angle$ 0.00	0.9462 $\angle$ −106.04	0.9462 $\angle$ 106.04
4	1	0.5903 $\angle$ 0.00	0.9560 $\angle$ −107.98	0.9560 $\angle$ 107.98
	2	0.2309 $\angle$ 0.00	0.9401 $\angle$ −104.70	0.9401 $\angle$ 104.70
	3	0.4387 $\angle$ 0.00	0.9354 $\angle$ −103.56	0.9354 $\angle$ 103.56
	4	0.0000 $\angle$ 0.00	0.9432 $\angle$ −105.41	0.9432 $\angle$ 105.41
	5	0.3463 $\angle$ 0.00	0.9386 $\angle$ −104.35	0.9386 $\angle$ 104.35
5	1	0.4764 $\angle$ 0.00	0.9400 $\angle$ −104.68	0.9400 $\angle$ 104.68
	2	0.1736 $\angle$ 0.00	0.9651 $\angle$ −109.57	0.9651 $\angle$ 109.57
	3	0.7043 $\angle$ 0.00	0.9751 $\angle$ −111.17	0.9751 $\angle$ 111.17
	4	0.5209 $\angle$ 0.00	0.9592 $\angle$ −108.55	0.9592 $\angle$ 108.55
	5	0.0000 $\angle$ 0.00	0.9681 $\angle$ −110.07	0.9681 $\angle$ 110.07